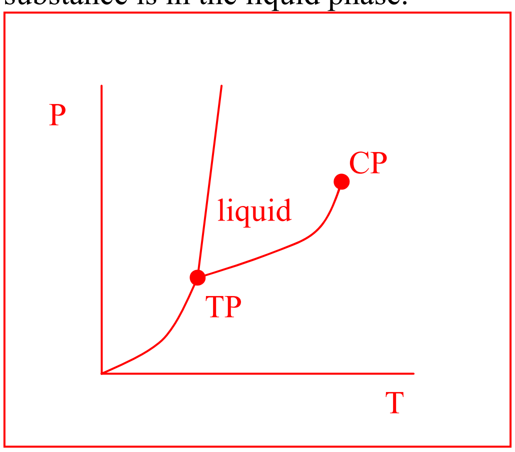
# P317 midterm solutions (k07)

1. Short answer questions:
   (a) State the 2nd law of thermodynamics (precisely!).
   The entropy of an isolated system cannot decrease.

   (b) What is the specific entropy change in a free expansion from specific volume $v_1$ to specific volume $v_2$?
   For a free expansion the heat flow and work are zero so du = 0. Then
   $T ds = du + P\,dv = P\,dv$ so $s_2 - s_1 = R \ln(v_2/v_1)$

   (c) What conditions must be met to have an equilibrium state?
   The pressure, temperature and chemical composition of the system must be static (unchanging in time); these correspond to mechanical, thermal and chemical equilibrium.

   (d) Draw the P versus T projection of the PVT surface for a substance that *contracts* upon freezing. Label the critical and triple points and indicate where the substance is in the liquid phase.



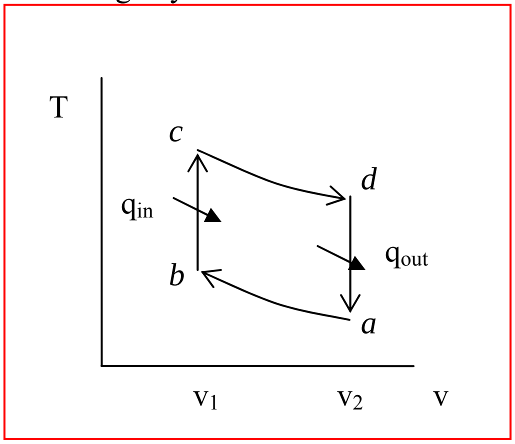

2. Consider the following equation of state, $(P-a)(v-b)=RT$, which describes a gas. The gas has constant specific heats $c_v$ and $c_P$.
   (a) Find coefficient of volume expansion for this gas.
   $\beta = (1/v)(\partial v/\partial T)_P = R/(v(P-a)) = (1-b/v)/T$

   (b) Find the specific entropy as a function of T and P.
   $ds = c_P\, dT/T - T(\partial v/\partial T)_P\, dP$ and $(\partial v/\partial T)_P = R/(P-a)$ so
   $s(P,T) = s_0 + c_P \ln(T/T_0) - R \ln((P-a)/(P_0-a))$

   (c) Find the Joule-Thomson coefficient $\mu$ for this gas.
   $\mu = (\partial T/\partial P)_h = -(\partial h/\partial P)_T / (\partial h/\partial T)_P$. The first partial can be derived from
   $T ds = dh - v\, dP = c_P\, dT - T(\partial v/\partial T)_P$ so $(\partial h/\partial P)_T = v - T(\partial v/\partial T)_P$.
   Then $\mu = -[v - T(\partial v/\partial T)_P]/c_P = -[v - (v-b)]/c_P = -b/c_P$

3. A monatomic ideal gas is taken through the reversible following reversible cycle:
(*a*-*b*) Adiabatic compression from volume $v_2$ to volume $v_1$
(*b*-*c*) Isochoric heating
(*c*-*d*) Adiabatic expansion from volume $v_1$ to volume $v_2$
(*d*-*a*) Isochoric cooling

(a) Draw this cycle in the T-V plane, labeling each of the points *a* through *d* and indicating any heat flows into or out of the system.




(b) Find the change in specific entropy of the gas in each of the processes in the *abcda* loop.

For *ab* and *cd* the change in entropy is zero (*reversible adiabatic*).
For *bc* and *da* there is no configuration work so the heat flows are equal to the changes in internal energy, namely $q_{in} = c_v(T_c-T_b)$ and $q_{out} = c_v(T_a-T_d)$.
The entropy can be found from $Tds = c_v\, dT + T(\partial P/\partial T)_v\, dv = c_v\, dT$ $(dv = 0)$ giving $s_c-s_b = c_v \ln (T_c/T_b)$ and $s_a-s_d = c_v \ln (T_a/T_d)$.

(c) Give a numerical value for the efficiency of the cycle when the ratio $v_2/v_1 = 5$.

By definition the efficiency is $\eta = w_{net} / q_{in} = (q_{in}+q_{out}) / q_{in}$. The heat flows take place at constant volume, so $\Delta q = c_v\, \Delta T$. Then $\eta = 1 + c_v (T_a-T_d) / [c_v (T_c-T_b)]$.
For an ideal gas in a reversible adiabatic process, $Tv^{\gamma-1}$ = constant, so
$T_a = T_b (v_1/v_2)^{\gamma-1}$ and $T_d = T_c (v_1/v_2)^{\gamma-1}$.
Then $\eta = 1 - (v_1/v_2)^{\gamma-1} = 1 - (1/5)^{2/3} = 0.658$.